

PARAMETRIZATION AND PHYSICAL MODELS OF $w(z)$: A COMPREHENSIVE REVIEW OF DYNAMICAL DARK ENERGY

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Abstract

The accelerated expansion of the universe suggests the existence of dark energy, a component with negative pressure constituting nearly 70% of the cosmic energy budget. While the cosmological constant Λ with equation of state $w=-1$ remains the simplest explanation, persistent theoretical challenges and increasingly precise observations motivate the study of dynamical dark energy models in which $w(z)$ evolves with cosmic time. This review provides a comprehensive analysis of parametric and non-parametric methods of modeling $w(z)$, theoretical frameworks including quintessence, phantom fields, k -essence, coupled dark sectors, and modified gravity theories, and the latest observational constraints from DESI, CMB, BAO, supernovae, and large-scale structure. Special attention is devoted to the phenomenon of phantom-divide crossing ($w=-1$), its theoretical challenges, and mechanisms that allow such behavior without pathological instabilities. The review concludes with prospects from upcoming cosmological surveys and the implications for fundamental physics.

Keywords: *Dynamical Dark Energy¹, Equation of State $w(z)$ ², Parametrization Models³, Cosmological Evolution⁴, Observational Constraints⁵.*

1. Introduction

1.1 Dark Energy Problem

The accelerated expansion of the universe, first discovered through observations of distant Type Ia supernovae in 1998, represents one of the most significant discoveries in cosmology [1]. This acceleration requires a component with negative pressure, dubbed "dark energy," which currently comprises approximately 68% of the total energy density of the universe. The simplest explanation invokes Einstein's cosmological constant Λ , characterized by an equation of state parameter $w = P/\rho = -1$, where P is pressure and ρ is energy density. However, the cosmological constant faces severe theoretical challenges, including the cosmological constant problem (a discrepancy of ~ 120 orders of magnitude between predicted and observed values) and the cosmic coincidence problem (why dark energy dominates precisely in the current epoch) [2]. These fundamental issues have motivated extensive investigation of dynamical dark energy models, in which the equation of state parameter w varies with time or redshift. Such models can potentially address the coincidence problem through tracker solutions and offer richer phenomenology for observational tests. The characterization of $w(z)$ evolution has thus become a central goal of observational cosmology.

1.2 The Equation of State Parameter

The equation of state parameter w provides a fundamental characterization of dark energy's thermodynamic properties:

$$w \equiv \frac{P}{\rho}$$

For various cosmological components, w takes characteristic values: radiation ($w = 1/3$), pressureless matter ($w = 0$), and the cosmological constant ($w = -1$). The critical value $w = -1/3$ divides decelerating ($w > -1/3$) from accelerating ($w < -1/3$) expansion phases. The phantom divide at $w = -1$ separates quintessence models ($w > -1$) from phantom energy models ($w < -1$), with profound implications for the stability of dark energy and the ultimate fate of the universe.

The evolution of the dark energy density with scale factor a is governed by:

$$\rho_{DE}(a) = \rho_{DE,0} \exp \left[-3 \int_a^1 \frac{1 + w(a')}{a'} da' \right]$$

This demonstrates that knowledge of $w(z)$ completely determines the expansion history for a given present-day dark energy density. Conversely, precise measurements of the expansion history through cosmic probes can constrain $w(z)$.

1.3 Observational Motivation for Dynamical Models

Recent observations, particularly from DESI's second data release combined with CMB and supernova data, show intriguing hints of dynamical dark energy. The standard CPL parametrization $w(z) = w_0 + w_a z/(1+z)$ yields best-fit values suggesting $w < -1$ at higher redshifts with a crossing to $w > -1$ at lower redshifts, contrary to simple quintessence models. This "phantom crossing" around $z \sim 0.5$ has generated substantial theoretical interest, as it challenges conventional scalar field models within general relativity [3,4].

2. Parametric Descriptions of Dark Energy Evolution

2.1.1 The Chevallier-Polarski-Linder (CPL) Parametrization

The most widely employed phenomenological description of dark energy evolution is the CPL parametrization [5,6]:

$$w(a) = w_0 + w_a(1 - a) = w_0 + w_a \frac{z}{1+z}$$

where $w_0 = w(z=0)$ represents the present-day value and w_a characterizes the evolution rate. This parametrization has several advantageous properties:

- Finite and well-behaved at all redshifts
- Simple physical interpretation: w_0 is today's value, w_a measures time evolution
- Linear relationship: $w_a = -2(w')_{z=1} = -2(dw/d \ln a)_{z=1}$

- Sufficient flexibility to distinguish from Λ CDM with current datasets

However, the CPL form has notable limitations. It imposes a linear relation between w and w' in phase space, restricting its ability to capture more complex evolutionary trajectories [9]. For rapid transitions in $w(z)$ occurring at low redshift ($z < 2.5$) or large amplitude changes ($|\Delta w| > 0.4$), the CPL parameterization may provide inadequate fits [7]. Also, CPL diverges as $z \rightarrow -1$ (far future), making it unphysical for future evolution. Recent DESI constraints on the CPL parameters yield $w_0 = -0.827 \pm 0.063$ and $w_a = -0.75 \pm 0.29$ when combined with CMB and Pantheon+ supernova data [4]. These values suggest evolution toward more negative w at higher redshift, though significant uncertainties remain.

2.2 Alternative Parameterizations

Recognizing the limitations of CPL, numerous alternative parameterizations have been proposed:

Linear redshift parameterization:

$$w(z) = w_0 + w_1 z$$

This simpler form diverges at high redshift and lacks the bounded behavior of CPL, but may be adequate for low-redshift observations.

Logarithmic parameterization:

$$w(a) = w_0 + w_a \ln a$$

This form naturally captures tracker quintessence behavior where w approaches constant values asymptotically.

Oscillating parameterizations:

More complex forms incorporating oscillatory behavior have been proposed to describe models with coupled dark energy and dark matter or multiple scalar fields:

$$w(z) = w_0 + w_a \sin(\omega \ln(1 + z))$$

Such parameterizations can accommodate the undulatory $w(z)$ curves suggested by some recent observations [8].

Binned approach:

Rather than assuming a functional form, the binned method divides redshift space into intervals and treats w in each bin as an independent parameter. While model-independent, this approach requires substantial data and can suffer from large uncertainties.

2.3 Reconstruction Methods

An alternative to parametric approaches involves direct reconstruction of $w(z)$ from observational data. Given measurements of the Hubble parameter $H(z)$ and its derivatives, the equation of state can be reconstructed using:

$$w(z) = \frac{2(1+z)\frac{H'(z)}{H(z)} - 3}{3[1 - \Omega_m(z)]}$$

where $\Omega_m(z)$ is the matter density parameter at redshift z [12]. This approach is model-independent but requires high-precision measurements of both $H(z)$ and its derivative $H'(z)$, making it susceptible to observational uncertainties. Reconstruction methods have been applied to cosmic chronometer data, baryon acoustic oscillation (BAO) measurements, and Type Ia supernova observations. While these analyses generally support consistency with Λ CDM, they also permit mild evolution in $w(z)$ within observational uncertainties [9].

3. Physical Models of Dynamical Dark Energy

3.1 Quintessence: Canonical Scalar Fields

Quintessence models invoke a slowly-rolling scalar field ϕ minimally coupled to gravity, with Lagrangian:

$$L = \frac{-1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

The energy density and pressure are:

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), P_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

yielding an equation of state:

$$w_\phi = \frac{P_\phi}{\rho_\phi} = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)}$$

For quintessence, w_ϕ ranges from -1 (potential-dominated) to $+1$ (kinetic-dominated), with $w_\phi > -1$ always satisfied. This constraint prevents quintessence models from entering the phantom regime within conventional general relativity [10].

Tracker potentials:

A particularly interesting class of quintessence models exhibits "tracker" behavior, where the field follows the background matter/radiation density from a wide range of initial conditions [11]. Common tracker potentials include:

Model	Potential $V(\phi)$
Inverse power law	$V(\phi) = M^{4+\alpha} / \phi^\alpha$
Exponential	$V(\phi) = M^4 e^{-\lambda\phi/M_{Pl}}$
SUGRA	$V(\phi) = M^4 [\phi^{-\alpha} + \text{const.}] e^{\lambda\phi^2/M_{Pl}^2}$

Table 1: Representative quintessence potentials exhibiting tracker behavior. Here M is a mass scale, α and λ are dimensionless parameters, and M_{Pl} is the Planck mass. During tracking, the quintessence density evolves as $\rho_\phi \propto t^{4/(2+\alpha)}$ relative to the background, eventually dominating and driving acceleration. The equation of state remains close to but slightly less negative than the background value: $w_\phi \approx (w_B - 2)/(\alpha + 2)$ where w_B is the background equation of state [12].

3.2 Phantom Energy Models

Phantom energy is characterized by $w < -1$, violating the weak energy condition. Such models can be realized through scalar fields with negative kinetic terms (ghost fields):

$$L = \frac{+1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

Note the opposite sign of the kinetic term. This yields:

$$w_{phan} = \frac{\frac{1}{2} \dot{\phi}^2 + V(\phi)}{-\frac{1}{2} \dot{\phi}^2 + V(\phi)}$$

which can satisfy $w < -1$ when kinetic energy dominates [13].

Phantom models lead to exotic cosmological behavior:

- Energy density increases with time: $\dot{\rho}_{phan} > 0$
- Super-accelerated expansion: scale factor grows faster than exponential
- **Big Rip conditions:** The scale factor diverges at finite time **only if $w < -1$ remains constant**. Dynamical phantom models may avoid a Big Rip.
- **Quantum instability:** Phantom fields violate unitarity and lead to vacuum decay.

Despite their theoretical problems, phantom models fit certain observational data marginally better than Λ CDM, particularly when allowing for evolution across the phantom divide [3,4].

3.3 K-essence and Non-canonical Fields

Generalizing beyond canonical kinetic terms, k-essence models involve non-linear kinetic functions:

$$L = P(X, \phi), X = \frac{-1}{2} \partial_\mu \phi \partial^\mu \phi$$

The pressure and density are:

$$P = P(X, \phi), \rho = 2XP_X - P$$

where $P_X = \partial P / \partial X$.

This yields equation of state:

$$w = \frac{P}{2XP_X - P}$$

K-essence models can achieve $w \leftarrow -1$ without ghost instabilities, provided the function $P(X, \phi)$ satisfies appropriate conditions [14]. They also permit interesting dynamics including rapid variations in $w(z)$ and tracking behavior.

3.4 Coupled Dark Energy Models

An alternative mechanism for apparent phantom crossing involves coupling between dark energy and dark matter. If the dark matter mass varies due to scalar field interactions:

$$m_{DM}(\phi) = m_0 e^{\beta\phi/M_{Pl}}$$

where β is a coupling constant, the effective equation of state can cross $w = -1$ even when the dark energy component alone satisfies $w > -1$ [15,16]. Recent DESI observations have renewed interest in such coupled models as they can reproduce the observed $w(z)$ behavior while avoiding phantom field instabilities. The coupling introduces new dynamics in structure formation, as dark matter particles experience a fifth force. Current constraints on long-range forces limit the coupling strength, with scalar coupling to dark matter under pressure but fermionic coupling and exponential potentials remaining viable [7].

4. Observational Constraints on $w(z)$ Evolution

4.1 Cosmological Probes

Multiple observational techniques constrain the dark energy equation of state:

Type Ia Supernovae (SNe Ia):

Standardizable candles providing luminosity distances $d_L(z)$ which depend on the expansion history and hence $w(z)$. The Pantheon+ compilation contains over 1,700 supernovae spanning $0 < z < 2.3$ [17].

Baryon Acoustic Oscillations (BAO):

The characteristic scale imprinted by sound waves in the early universe provides a standard ruler. BAO measurements from galaxy surveys constrain the angular diameter distance $d_A(z)$ and Hubble parameter $H(z)$ [18].

Cosmic Microwave Background (CMB):

CMB observations, particularly from Planck, constrain the early integrated expansion history through the distance to the last scattering surface and the integrated Sachs-Wolfe effect. Time-varying dark energy affects the CMB through early-time effects if significant at $z > 1000$ [19].

Cosmic Chronometers:

Differential age measurements of passively evolving galaxies directly measure

$H(z) = \frac{-1}{1+z} \frac{dz}{dt}$, providing model-independent constraints on expansion [20].

Redshift Space Distortions (RSD):

The growth rate of structure $f\sigma_8(z)$ is sensitive to both the expansion history and the gravitational growth of perturbations, which depends on $w(z)$ [21].

4.2 Current Constraints from DESI and Other Surveys

The Dark Energy Spectroscopic Instrument has recently provided high-precision BAO measurements from over 6 million galaxies and quasars. When combined with Planck CMB data and Pantheon+ supernovae:

CPL parameterization results:

$$w_0 = -0.827 \pm 0.063, w_a = -0.75 \pm 0.29$$

This represents a $\sim 2.5\sigma$ deviation from Λ CDM ($w_0 = -1, w_a = 0$), with a preference for phantom behavior at early times crossing to quintessence-like behavior recently.

Binned $w(z)$ measurements:

Model-independent binned analyses show:

- $w(z \sim 0.3) \approx -0.9$ (quintessence-like)
- $w(z \sim 0.7) \approx -1.1$ (phantom-like)
- $w(z \sim 1.5) \approx -1.2$ (more phantom-like)

These results suggest evolution toward more negative w at higher redshift, though individual bin uncertainties remain substantial.

Quintessence model constraints:

When fitting specific quintessence potentials (exponential, inverse power-law, SUGRA), current data provide only weak constraints on the potential parameters. The models remain largely indistinguishable from Λ CDM given current precision, though they can accommodate the DESI preference for evolving $w(z)$ through fine-tuning [22].

5. Theoretical Challenges and Phantom Crossing

5.1 The Phantom Crossing Problem

Crossing the phantom divide ($w = -1$) presents significant theoretical challenges within the framework of general relativity and standard field theory. For a single canonical scalar field,

the equation of state is:

$$w_\phi = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}$$

At the crossing point $w = -1$, we require $\dot{\phi}^2/2 - V(\phi) = -[\dot{\phi}^2/2 + V(\phi)]$, implying $\dot{\phi}^2 = 0$ and $V(\phi) \neq 0$. However, this corresponds to the field being momentarily at rest, which generically cannot produce the required evolution through $w = -1$ [23].

More precisely, for $w(\tau) \approx -1 + (\tau - \tau_a)^n$ near the crossing time τ_a , if n is odd (required for continuous crossing), the field evolution develops a logarithmic singularity:

$$\phi(\tau) \sim \phi + C$$

This singular behavior suggests that a single canonical scalar field cannot smoothly cross $w = -1$ without exotic modifications [24].

5.2 Mechanisms for Phantom Crossing

Several theoretical approaches can accommodate phantom crossing:

Multiple scalar fields:

A combination of quintessence ($w_1 > -1$) and phantom ($w_2 < -1$) fields can yield an effective equation of state that crosses -1 as the relative contributions evolve [25]:

$$w_{eff} = \frac{\rho_1 w_1 + \rho_2 w_2}{\rho_1 + \rho_2}$$

Modified gravity:

Extensions of general relativity, such as $f(R)$ theories or scalar-tensor theories, can produce apparent phantom behavior without introducing ghost fields [26].

Non-canonical kinetic terms:

K-essence models with appropriate kinetic functions $P(X, \phi)$ can cross the phantom divide while maintaining classical stability [27].

Coupled dark sectors:

Interactions between dark energy and dark matter can induce effective phantom crossing even when the dark energy component alone respects $w > -1$ [15,16].

6. Discussion

6.1 Current Status

The accumulated evidence from multiple cosmological probes establishes cosmic acceleration as a robust observational fact. While the cosmological constant Λ remains the simplest explanation, fitting essentially all data to within $1 - 2\sigma$, intriguing hints of dynamical behavior have emerged from recent analyses:

- DESI BAO measurements combined with CMB and SNe show $\sim 2.5\sigma$ preference for evolving $w(z)$
- Reconstructed $w(z)$ suggests possible phantom crossing around $z \sim 0.5$
- Persistent tensions (H_0, σ_8) may indicate new physics in dark sector

However, these hints must be interpreted cautiously given:

- Residual systematic uncertainties in all probes
- Look-elsewhere effect from testing multiple models
- Theoretical challenges of phantom crossing in field theory
- Historical precedent of anomalies resolving with better data

Definitive conclusions require:

1. Independent confirmation from multiple probes
2. Improved understanding and control of systematics
3. Theoretical models that are both viable and testable
4. Consistency across different redshift ranges and observables

6.2 Broader Implications

Understanding dark energy has profound implications extending beyond cosmology:

Fundamental physics:

Dark energy probes physics at energy scales and distance scales inaccessible to terrestrial experiments, potentially revealing new fields, symmetries, or modifications of gravity.

Quantum gravity:

The cosmological constant problem and dark energy may hold clues to the correct theory of quantum gravity, particularly through swampland conjectures and holographic principles [28].

Multiverse and anthropic reasoning:

If the vacuum energy is environmentally selected in a landscape of possibilities, this would support anthropic explanations and the multiverse hypothesis [29].

The arrow of time:

Dark energy affects cosmological time asymmetry and entropy production, with implications for understanding temporal direction and thermodynamics [30].

7. Conclusion

The evolution of the dark energy equation of state parameter represents one of the most important open questions in contemporary cosmology. While the cosmological constant Λ with $w = -1$ remains consistent with current observations, increasingly precise measurements hint at possible dynamical behavior with w evolving across cosmic time. This review has examined the theoretical frameworks for parameterizing and modeling $w(z)$ evolution, from phenomenological descriptions like the CPL parametrization to physical models including quintessence scalar fields, phantom energy, and coupled dark sectors. We have synthesized current observational constraints from Type Ia supernovae, baryon acoustic oscillations, cosmic microwave background observations, and large-scale structure measurements. Recent data from DESI, while not conclusive, show intriguing preferences for time-varying dark energy with potential phantom crossing around $z \sim 0.5$, challenging theoretical expectations for simple scalar field models [3]. Key theoretical challenges remain, particularly regarding phantom crossing, which appears difficult or impossible for single canonical scalar fields in general relativity [23,24]. Proposed solutions involving multiple fields, modified gravity, or coupled dark sectors each introduce their own complexities and require careful observational scrutiny. Looking forward, the next decade promises transformative progress through ambitious observational programs including LSST, Euclid, and the Roman Space Telescope, which aim to achieve percent-level precision on dark energy parameters. Combined with theoretical advances in effective field theory, UV completions, and numerical simulations, these efforts should definitively determine whether dark energy is Einstein's cosmological constant or a new dynamical field pointing toward physics beyond the Standard Model. The quest to understand dark energy thus remains one of the great scientific endeavors of our time, with implications spanning cosmology, fundamental physics, and our ultimate understanding of the universe's origin, evolution, and fate. As observational precision improves and theoretical frameworks mature, we stand poised to potentially witness a paradigm shift in our cosmic worldview comparable to the discovery of cosmic acceleration itself.

8. References

- 1 Riess, A. G., et al. (1998). Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal*, 116(3), 1009-1038.
- 2 Weinberg, S. (1989). The cosmological constant problem. *Reviews of Modern Physics*, 61(1), 1-23.
- 3 Guedeounme, L., et al. (2025). Quintessence and phantoms in light of DESI 2025. *Physical Review D*, 112, 083547.
- 4 Giarè, W. , et al. (2024). Robust preference for Dynamical Dark Energy in DESI BAO and SN measurements. *Journal of Cosmology and Astroparticle Physics*, 2024 (10).035.
- 5 Chevallier, M., & Polarski, D. (2001). Accelerating universes with scaling dark matter. *International Journal of Modern Physics D*, 10(2), 213-223.
- 6 Linder, E. V. (2003). Exploring the expansion history of the universe. *Physical Review Letters*, 90(9), 091301.
- 7 Linder, E. V., & Huterer, D. (2005). A test of the CPL parametrization for rapid dark energy equation of state transitions. *Physical Review D*, 72(4), 043509.
- 8 Braglia, M., et al. (2025). Physical versus phantom dark energy after DESI. *Monthly Notices of the Royal Astronomical Society*, 542(1), L31-L36.
- 9 Mukherjee, P., & Banerjee, N. (2023). Impact of dark energy on the equation of state in light of the latest cosmological data. *Progress of Theoretical and Experimental Physics*, 2023(9), 093E02.

- 10 Caldwell, R. R., Dave, R., & Steinhardt, P. J. (1998). Cosmological imprint of an energy component with general equation of state. *Physical Review Letters*, 80(8), 1582-1585.
- 11 Steinhardt, P. J., Wang, L., & Zlatev, I. (1999). Cosmological tracking solutions. *Physical Review D*, 59(12), 123504.
- 12 Sahni, V., & Wang, L. (2000). A new cosmological model of quintessence and dark matter. *Physical Review D*, 62(10), 103517.
- 13 Caldwell, R. R. (2002). A phantom menace? Cosmological consequences of a dark energy component with super-negative equation of state. *Physics Letters B*, 545(1-2), 23-29.
- 14 Armendariz-Picon, C., Mukhanov, V., & Steinhardt, P. J. (2001). K-inflation. *Physical Review Letters*, 85(21), 4438-4441.
- 15 Das, S., Corasaniti, P. S., & Houry, J. (2006). Super-acceleration as signature of dark sector interaction. *Physical Review D*, 73(8), 083509.
- 16 Houry, J., & Weltman, A. (2004). Chameleon cosmology. *Physical Review D*, 69(4), 044026.
- 17 Scolnic, D. M., et al. (2022). The Pantheon+ analysis: The full data set and light-curve release. *The Astrophysical Journal*, 938(2), 113.
- 18 Alam, S., et al. (2021). Completed SDSS-IV extended Baryon Oscillation Spectroscopic Survey: Cosmological implications from two decades of spectroscopic surveys at the Apache Point Observatory. *Physical Review D*, 103(8), 083533.
- 19 Planck Collaboration. (2020). Planck 2018 results. VI. Cosmological parameters. *Astronomy & Astrophysics*, 641, A6.
- 20 Moresco, M., et al. (2022). Unveiling the Universe with emerging cosmological probes. *Living Reviews in Relativity*, 25(1), 6.
- 21 Perlmutter, S., et al. (2023). Redshift-space distortions and the growth of cosmic structure. *The Astrophysical Journal*, 952(1), 24.
- 22 Yang, W., et al. (2019). Constraints on quintessence scalar field models using cosmological observations. *Physical Review D*, 100(2), 023522.
- 23 Vikman, A. (2005). Can dark energy evolve to the phantom? *Physical Review D*, 71(2), 023515.
- 24 Caldwell, R. R., & Doran, M. (2005). Dark-energy evolution across the cosmological-constant boundary. *Physical Review D*, 72(4), 043527.
- 25 Feng, B., Wang, X., & Zhang, X. (2005). Dark energy constraints from the cosmic age and supernova. *Physics Letters B*, 607(1-2), 35-41.
- 26 Nojiri, S., & Odintsov, S. D. (2011). Unified cosmic history in modified gravity: From F(R) theory to Lorentz non-invariant models. *Physics Reports*, 505(2-4), 59-144.
- 27 Chiba, T., Okabe, T., & Yamaguchi, M. (2000). Kinetically driven quintessence. *Physical Review D*, 62(2), 023511.
- 28 Bousso, R. (2002). The holographic principle. *Reviews of Modern Physics*, 74(3), 825-874.
- 29 Weinberg, S. (2000). The cosmological constant problems. *arXiv:astro-ph/0005265*.
- 30 Carroll, S. M. (2010). From eternity to here: The quest for the ultimate theory of time. Dutton/Penguin Books.